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Thermal and wear analysis of an elastic beam in sliding contact

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Abstract

The heat conduction and wear of a cantilever beam in contact with a moving object at the free end is studied. The problem is formulated as a system of coupled nonlinear differential equations. A finite element algorithm, which incorporates an implicit time integration scheme, is developed to solve the problem. Numerical results are presented and discussed. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The paper presents the study of thermal conduction and wear of a cantilever beam, which is in frictional contact with a moving rigid part. The interest lies in the evolution of the wear of the beam's contacting end as a result of the frictional shear stress and the associated heat generation and conduction. Situations where parts and components come into contact, which is accompanied by heat generation and wear, abound in industry. An obvious example is the car braking system where both frictional heat generation and wear are present and are important to understand and control. Modeling and analysis of the fully three-dimensional dynamic problems of thermoelastic frictional contact with wear is complicated, and not well understood. Kennedy and Ling (1974) presented an axisymmetric finite element model to study the heat conduction and wear of high-energy disk brakes. A heat source element representing friction and a wear criterion similar to that in plastic flow were used. In the numerical study presented by Azarkhin et al. (1989), governing integral equations together with Green functions were used to model the mechanical and thermal behavior of two half-planes during frictional sliding, but wear was excluded in this study. Along this line of research, Johansson and Klarbring (1993) used an iterative finite element method to deal with the coupled thermomechanical problem. As in Azarkhin et al. (1989), wear was not considered. Later, Johansson (1993) added the effects of wear in a numerical study for sliding contact between two elastic half-planes using the integral equation method. Recently, Andrews et al. (1997a,b) presented mathematical studies in which they

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established the existence of solutions for the thermoviscoelastic contact problem. Strömberg et al. (1996) derived a generalized standard model for contact friction and wear from the second law of thermodynamics and the principle of virtual work. Recently, Strömberg (1999) presents a numerical treatment of two-dimensional thermoelastic wear analysis in which Signorini contact condition, Coulomb friction, and Archard's law of wear are included. In Andrews et al. (1997a,b) and Kennedy and Ling (1974), examples with simpler one-dimensional settings are considered. It is believed that the one-dimensional setting, that is considered here, may be easy to set up experimentally so that the various physical parameters entering the problem can be measured accurately with relative ease.

In this paper, the one-dimensional problem in which the elasticity problem is coupled with the thermal problem and the one for wear is considered. The model consists of the equation of heat conduction for the temperature field and the rate version of the law of Archard (1953) for wear. The inertial term in the equation of motion for the Euler–Bernoulli beam is assumed negligible, and thus the equation can be integrated to obtain the contact pressure. The frictional heat generated at the contacting end is taken into account, and so is the heat lost to the environment from the lateral face of the beam. Two versions of the problem were analyzed recently in Gu et al. (2000) where the existence and regularity of weak solutions were proved.

The model is described in Section 2. It is set in a variational form in Section 3 and the finite element method applied to it. Since the problem is nonlinear, an iteration method is employed. Two examples of the numerical procedure are presented in Section 4. In the first one, the rigid object is fixed in space and consequently the contact pressure varies in time. A study of the temperature at the contact point as a function of time for three different values of the friction coefficient is presented. In the second example, the contact pressure is maintained at a constant value by the motion of the rigid object. The conclusions of this study are presented in Section 5.

2. Mathematical description of the problem

The problem considered, shown in Fig. 1, describes an elastic beam which is deflected by the frictional force developed between the beam and a rigid body moving at a constant speed V_0. The rigid body may be a rigid grinding wheel turning at a constant angular speed. To be investigated in this paper are the transient temperature distribution in the beam, wear behavior at the end of the beam, and the deformation of the

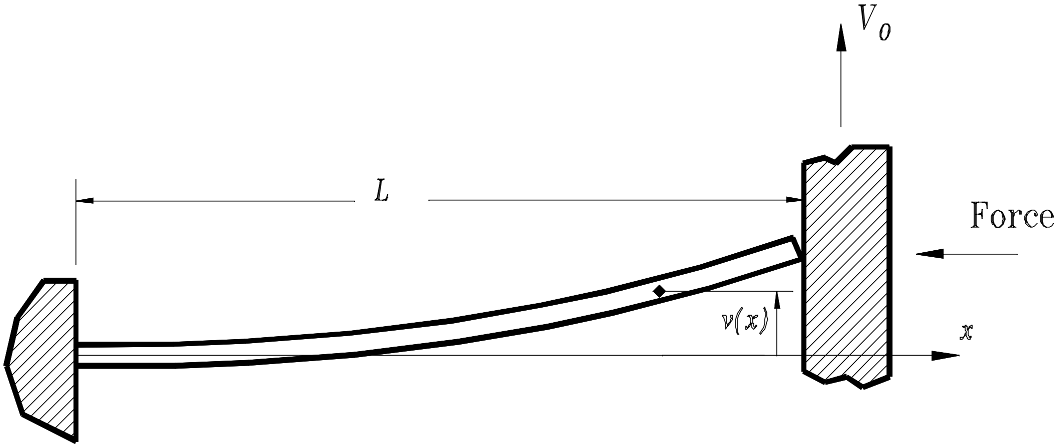

Fig. 1. An elastic beam deformed by frictional force from a moving rigid object.

beam. For simplicity, it is assumed that the temperature of the rigid body is constant and does not wear during the process.

2.1. Heat conduction in the beam

The mathematical description of the heat flow in the beam, shown in Fig. 2, is

$$KT_{,xx} - hr(T - T_{\mathrm{a}}) = \rho c\dot{T}, \quad 0 < x < L, \;\; t > 0, \tag{1}$$

$$T(0,t) = T_{\mathrm{a}}, \quad t > 0, \tag{2}$$

$$-KT_{,x}(L,t) = \rho c T_L \dot{w} - \mu p V_0 + h_{\mathrm{s}}(T_L - T_{\mathrm{s}}), \quad t > 0, \tag{3}$$

$$T(x,0) = T_{\mathrm{a}}, \quad 0 < x < L, \tag{4}$$

where $T(x,t)$ is the temperature in the beam at time t; the dot above a variable indicates a time derivative; K, the thermal conductivity of the material; h, the heat transfer coefficient between the beam and the environment; r, the ratio of perimeter b to the cross-sectional area A of the beam; T_{a}, the ambient temperature; ρ, the density of the material, c, the heat capacity of the material; μ, the coefficient of kinetic friction; p, the current end pressure at $x = L$; T_L, the temperature at the right end; T_{s}, the temperature of the sliding object; h_{s}, the heat transfer coefficient between the beam and the sliding object; and $\dot{w}$, the rate of wear. For convenience, V_0 represents the absolute value of the speed of the sliding object. The temperature rise in the beam is the result of the friction given by the second term on the right-hand side of Eq. (3). The temperature at the left end is equal to T_{a} at all times. In this one-dimensional problem, it is assumed that the heat also dissipates into the environment through the lateral surface of the beam as well as into the sliding object via convection. Johansson and Klarbring (1993) employ a pressure-dependent contact conductance in place of h_{s} to account for the heat conduction between two sliding objects. It is assumed that the debris leaving the system also carry away some energy given by the first term on the right-hand side of Eq. (3). This mechanism has been used by Johansson (1993). The sliding object is assumed rigid throughout.

2.2. Wear analysis

To determine the amount of wear at the right end during sliding Archard's law of wear is used:

$$\dot{w} = k_{\mathrm{w}} p V_0, \tag{5}$$

where k_{w} is the wear constant. Therefore, let $\gamma = (\rho c k_{\mathrm{w}} T_L - \mu)V_0$, Eq. (3) becomes

$$-KT_{,x}(L,t) = \gamma p + h_{\mathrm{s}}(T_L - T_{\mathrm{s}}). \tag{6}$$

For most engineering materials, $\rho c k_{\mathrm{w}} T_L \ll \mu$; that is, the energy loss due to wear is negligibly small in comparison with the heat generated by friction. Thus, $\gamma \approx -\mu V_0$.

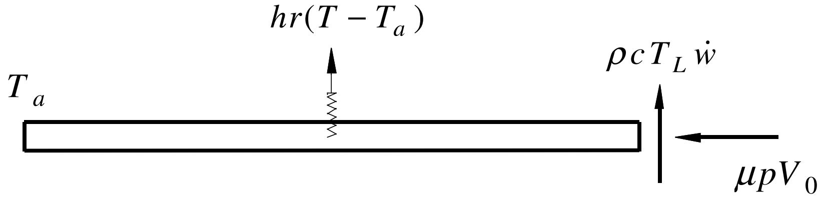

Fig. 2. Engorge flows in and out of the beam.

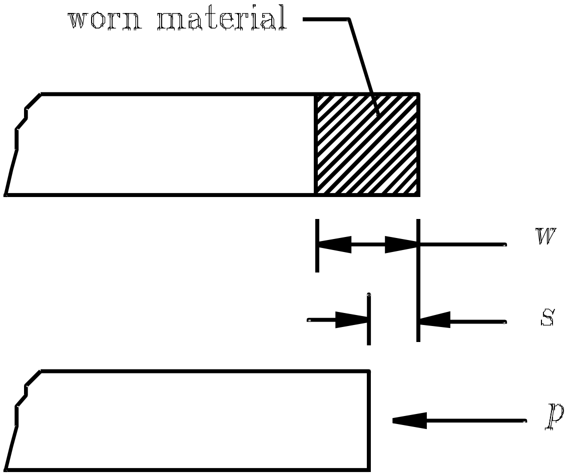

Fig. 3. Definition of the parameter β.

The beam is shortened because of the wear. This is illustrated in Fig. 3. It is postulated that the reduction in length, s, is proportional to the amount of wear, w. That is,

$$s = \beta w, \tag{7}$$

where β is the proportionality constant. It is assumed that w measures the actual amount of removed material, while s measures the equivalent shortening of the beam. When $\beta = 1$, the worn material has been removed completely and the contact surface is as if no wear has taken place, although the beam is shorter by $w = s$. When $\beta < 1$ scratching or ploughing takes place on the surface and it is not only shorter but also worn. Therefore, β measures the type of wear of the end.

2.3. Mechanics and contact pressure

From the Euler–Bernoulli beam theory, the axial stress at a point (x, y) in the beam is given by

$$\sigma_x(x, y, t) = -\frac{M(x, t)y}{I_z} - p(t), \tag{8}$$

where $M(x, t)$ is the bending moment; y, the distance from the point to the centroidal axis or the x-axis; I_z, the moment of inertia of the cross-sectional area about the z-axis; and p, the contact pressure as defined previously. In this problem, the bending moment is given by the linear relation below as a result of the end shear force μpA,

$$M(x, t) = \mu pA(L - x). \tag{9}$$

Following the stress–strain relation, the axial strain ε_x in the beam is written as

$$\varepsilon_x = -\frac{\mu pA(L - x)y}{EI_z} - \frac{p}{E} + \alpha(T - T_a), \tag{10}$$

where E is the modulus of elasticity and α is the coefficient of thermal expansion. The displacement along the x-axis can be found by setting $y = 0$ and integrating the above with respect to x from x to L. This yields

$$u(x) = u(L) + \frac{p(L-x)}{E} - \alpha \int_x^L (T - T_\mathrm{a})\,\mathrm{d}x. \tag{11}$$

Two types of boundary conditions are considered in this study. In the first type of boundary condition, the end pressure is assumed constant. In this case, the sliding object moves toward the left as the beam is worn and the x-displacement $u(L)$ is not known. In the second case, once the pressure is applied the sliding object is kept stationary in the x-direction. Therefore, the contact pressure varies with time while the x-displacement at the right end of the beam is given as follows,

$$u(L) = -\left(\frac{p_0 L_0}{E} - s\right), \tag{12}$$

where p_0 is the initial applied pressure and L_0 is the original length of the beam. Using Eq. (12) in Eq. (11), the contact pressure can be solved by using the fact that $u(0) = 0$. Thus,

$$p(t) = p_0 \frac{L_0}{L} - E\frac{s}{L} + \frac{\alpha E}{L} \int_0^L (T - T_\mathrm{a})\,\mathrm{d}x. \tag{13}$$

Here, the second term accounts for the pressure relief due to wear while the third term represents the pressure increase due to temperature surge in the beam. By using Eqs. (7) and (13), Archard's law of wear, Eq. (5), can be written as follows:

$$\dot{w} + \frac{k_\mathrm{w} V_0 E\beta}{L} w - \frac{k_\mathrm{w} V_0 \alpha E}{L} \int_0^L T\,\mathrm{d}x = k_\mathrm{w} V_0 \left(p_0 \frac{L_0}{L} - \frac{\alpha E}{L} \int_0^L T_\mathrm{a}\,\mathrm{d}x\right). \tag{14}$$

This equation together with Eqs. (1)–(4) will be used to solve the problem when the contact pressure is allowed to vary. Once the contact pressure is determined, the deflection of the beam, $v(x)$, is given by

$$v(x) = \frac{\mu p A}{6EI_z} x^2 (3L - x). \tag{15}$$

3. Numerical treatment

The weak form of the problem given in Eqs. (1)–(4) can be written as follows after using Eq. (6),

$$\int_0^L (\rho c \dot{T} \psi_j + K T_{,x} \psi_{j,x} + hrT\psi_j)\,\mathrm{d}x + [\gamma p - h_\mathrm{s}(T_L - T_\mathrm{s})]\psi_{jL} = \bar{q}_0 \psi_{j0} + \int_0^L hrT_\mathrm{a}\psi_j\,\mathrm{d}x, \tag{16}$$

where ψ_j is a test function, $\psi_{jL} = \psi_j(L)$, $\psi_{j0} = \psi_j(0)$, and $\bar{q}_0$ is the heat flux at $x = 0$. To discretize this equation, let N be the total number of nodes used and

$$T = \psi_k T_k, \tag{17}$$

where the indicial summation convention is implied here and subsequently. After using Eq. (13), Eq. (16) becomes,

$$C_{jk}\dot{T}_k + \left(B_{jk} + \gamma\psi_{jL} D_k + h_\mathrm{s}\psi_{jL}\delta_{kN}\right) T_k - \frac{\beta\gamma E}{L}\psi_{jL} w = R_j, \tag{18}$$

where, δ_{kN} is the usual Kronecker delta, and

$$C_{jk} = \int_0^L \rho c \psi_j \psi_k\,\mathrm{d}x,$$

$$B_{jk} = \int_0^L (K\psi_{j,x}\psi_{k,x} + hr\psi_j\psi_k)\,\mathrm{d}x, \tag{19}$$

$$D_k = \frac{\alpha E}{L}\int_0^L \psi_k\,\mathrm{d}x,$$

$$R_j = \bar{q}_0\psi_{j0} + \int_0^L hrT_\mathrm{a}\psi_j\,\mathrm{d}x - \psi_{jL}\left(\gamma\left(p\frac{L_0}{L} - \alpha ET_\mathrm{a}\right) + h_\mathrm{s}T_\mathrm{s}\right).$$

Eq. (14) can also be discretized by using Eq. (17). Hence,

$$\dot{w} + \frac{k_\mathrm{w}V_0\beta E}{L}w - k_\mathrm{w}V_0D_kT_k = k_\mathrm{w}V_0\left(p_0\frac{L_0}{L} - \alpha ET_\mathrm{a}\right). \tag{20}$$

Thus, it is readily seen that for the case of varying contact pressure, Eqs. (18) and (20) have to be solved simultaneously.

In the case wherein the applied pressure is held constant and the sliding rigid body moves to the left as the beam is being worn, the last term on the left-hand side of Eq. (18) is absent and the term on the right becomes

$$R_j = \bar{q}_0\psi_{j0} + \int_0^L hrT_\mathrm{a}\psi_j\,\mathrm{d}x - (\gamma p_0 - h_\mathrm{s}T_\mathrm{s})\psi_{jL}. \tag{21}$$

Therefore, Eq. (18), now without w, can be solved alone for the nodal temperatures. Since the length of the beam reduces with time, the upper limit of the integrals in the above is not known. A new variable defined below may be used to overcome this difficulty when performing the integration with respect to x,

$$\xi = \frac{x}{L}.$$

As a result, the upper limit becomes 1 and the length L precedes every integral above. Note that $L = L_0 - \beta w$.

An implicit scheme shown below is used to integrate the equations in time:

$$\frac{\Delta T_k}{\Delta t} = \zeta\dot{T}_k^{n+1} + (1-\zeta)\dot{T}_k^n,$$

where the superscript indicates the time step, Δt is the time-step size, and $0 < \zeta < 1$. Thus,

$$\dot{T}_k^{n+1} = \frac{1}{\zeta}\left[\frac{\Delta T_k}{\Delta t} - (1-\zeta)\dot{T}_k^n\right].$$

At time $t = (n+1)\,\Delta t$, Eq. (18) becomes

$$\left[\frac{1}{\Delta t}C_{jk} + \zeta(B_{jk} + \gamma\psi_{jL}D_k)\right]\Delta T_k - \frac{\zeta\beta\gamma E}{L^n}\psi_{jL}\Delta w = \zeta R_j^{n+1} + (1+\zeta)C_{jk}\dot{T}_k^n - \zeta(B_{jk} + \gamma\psi_{jL}D_k)T_k^n + \frac{\zeta\beta\gamma E}{L^n}\psi_{jL}w^n. \tag{22}$$

Similarly, Eq. (20) may be discretized in time resulting in

$$\left(\frac{1}{\Delta t} + \zeta\frac{k_\mathrm{w}V_0\beta E}{L^n}\right)\Delta w - \zeta k_\mathrm{w}V_0D_k\Delta T_k = \zeta k_\mathrm{w}V_0\left(p_0\frac{L_0}{L^n} - \alpha ET_\mathrm{a}\right) + (1-\zeta)\dot{w}^n - \zeta\frac{k_\mathrm{w}V_0\beta E}{L^n}w^n + \zeta k_\mathrm{w}V_0D_kT_k^n. \tag{23}$$

Since $L = L_0 - \beta w$, an iteration method is needed to solve this nonlinear system. In the first iteration at the current time step, the variables in Eqs. (22) and (23) take the converged values from the previous time step. From then on, they are updated using the newly calculated increments. At the end of each iteration the following errors must be checked for convergence using updated values at $t = (n+1)\,\Delta t$,

$$\varepsilon_T = \|R_j^{\mathrm{err}}\|,$$

$$\varepsilon_{\mathrm{w}} = \left| k_{\mathrm{w}} v_0 \left(p_0 \frac{L_0}{L^{n+1}} - \alpha E T_{\mathrm{a}} \right) - \left(\dot{w}^{n+1} + \frac{k_{\mathrm{w}} v_0 \beta E}{L^{n+1}} w^{n+1} - k_{\mathrm{w}} v_0 D_k T_k^{n+1} \right) \right|,$$

where $\|\cdot\|$ indicates a vector norm, and

$$R_j^{\mathrm{err}} = R_j^{n+1} - \left[C_{jk} \dot{T}_k^{n+1} + (B_{jk} + \gamma \psi_{jL} D_k) T_k^{n+1} - \frac{\beta \gamma E}{L^{n+1}} \psi_{jL} w^{n+1} \right].$$

No further iteration is needed if the errors are smaller than a preset tolerance.

4. Numerical examples and discussion

The elastic steel beam considered is 0.4 m in length and has a 5 mm^2 cross-section. The physical properties for the steel are: $E = 200$ GPa, $\rho = 7800\,\mathrm{kg/m^3}$, $c = 460\,\mathrm{J/kg\,°C}$, $K = 47$ W/m °C, $\alpha = 1.2 \times 10^{-5}/°\mathrm{C}$. The environment parameters used are: $T_{\mathrm{a}} = 20\,°\mathrm{C}$, $h = 200$ $\mathrm{W/m^2\,°C}$, $h_{\mathrm{s}} = 500\,\mathrm{W/m^2\,°C}$. The initial applied pressure is $p_0 = 1.0\,\mathrm{MPa}$ and the velocity of the sliding object is $V_0 = 10$ m/s. The value chosen for h_{s} is equal to the value of contact conductance for steel at a contact pressure of 1 MPa used in Johansson and Klabring (1993) and Johansson (1993). Note that for the steel, the yield stress is 200 MPa while the melting point is 1500°C.

4.1. Case 1: varying contact pressure

Fig. 4 displays the temperatures at the contact end of the beam using different values of the coefficient of kinetic friction. The calculation stops when the wear rate becomes zero, i.e. the contact pressure decreases to zero. The time it takes to reach that point is the wear life of the beam, the time needed to wear out the amount permitted of the beam. The number within the parentheses in the legend represents the wear life of each case. It is seen from the figure that the wear life grows longer with greater value of μ, because a greater value of μ generates more heat, which in turn causes the beam to expand. Further, the end-temperature T_L at the end of the wear life is in the order of 10^4°C if $\mu = 0.4$ is used. The wear constant used for Fig. 4 and the remaining figures in Case 1 is $k_{\mathrm{w}} = 1.0 \times 10^{-12}$ Pa^{-1} (Johansson, 1993). It is found that the end temperature T_L well exceeds the melting point if $k_{\mathrm{w}} = 1.0 \times 10^{-13}\,\mathrm{Pa}^{-1}$ is used. On the other hand, the wear life is less than 1.0 s if $k_{\mathrm{w}} = 1.0 \times 10^{-11}\,\mathrm{Pa}^{-1}$ is used. It is seen that the model is sensitive to the value of k_{w}. All three curves in Fig. 4 show a 10% decrease after reaching the maximum temperature. Fig. 5 shows the evolving contact pressures and the cumulative wear for various values of μ. For $\mu = 0.1$ and 0.2, the contact pressure decreases as time elapses. For $\mu = 0.3$, it first increases to a maximum, then decreases to zero. As seen in Eq. (13) the change in the contact pressure stems from the temperature increase and the wear of the beam. For $\mu = 0.3$, the effect of temperature increase on the contact pressure apparently surpasses that of wear in the first part of the wear life. So far, the parameter β has been set equal to 1.0. Figs. 6 and 7 explore the effect of the wear parameter β on the temperature, contact pressure, and wear, in which $\mu = 0.2$ is used. The case of $\beta = 0.5$ is not presented because the end-temperature exceeded the melting point of steel. The contact pressure for the case of $\beta = 0.6$ in Fig. 7 increases substantially before it vanishes. The contact

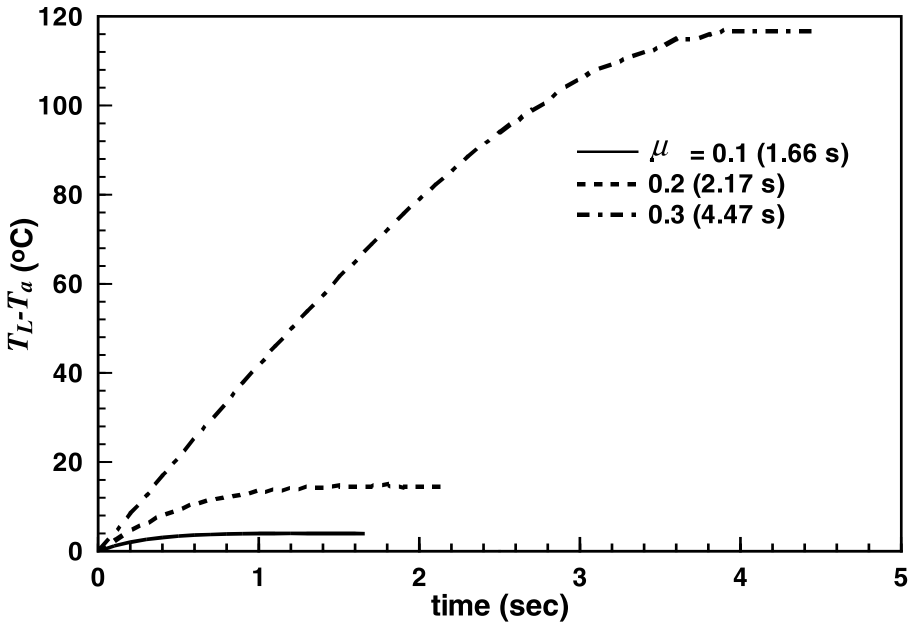

Fig. 4. End-temperature change $T_L - T_a$ versus time using various values of μ.

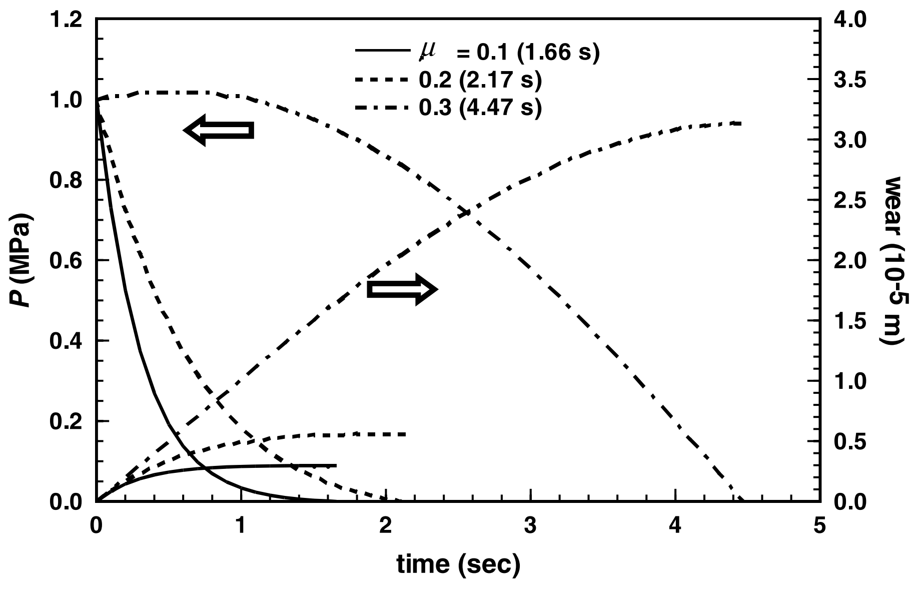

Fig. 5. Contact pressure and wear versus time using various values of μ.

pressure decreases monotonically for the cases with greater values of β. The deflection of the beam can be calculated with ease from Eq. (15) once the contact pressure is found. Thus, the deflection is not presented here.

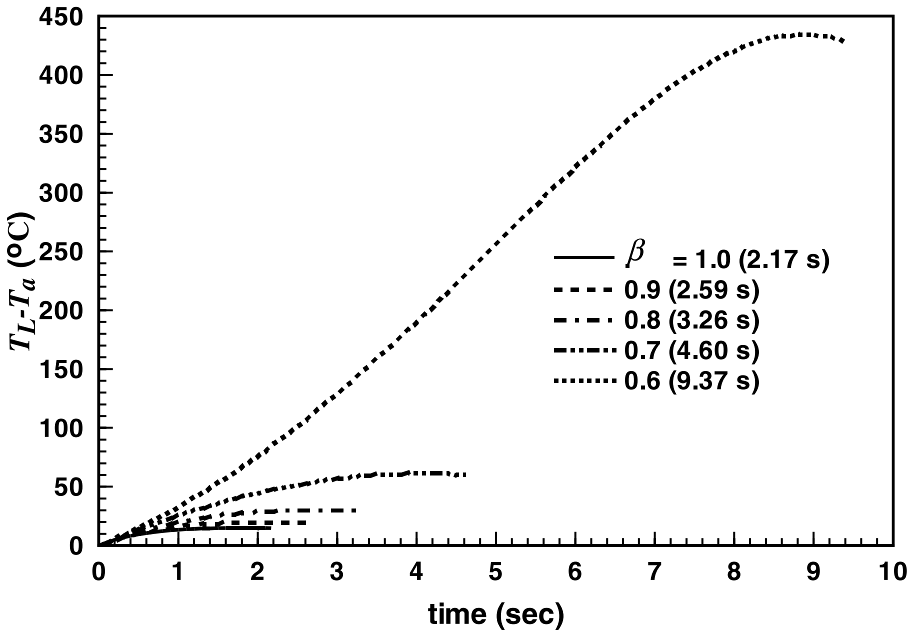

Fig. 6. End-temperature change $T_L - T_a$ versus time using various values of β.

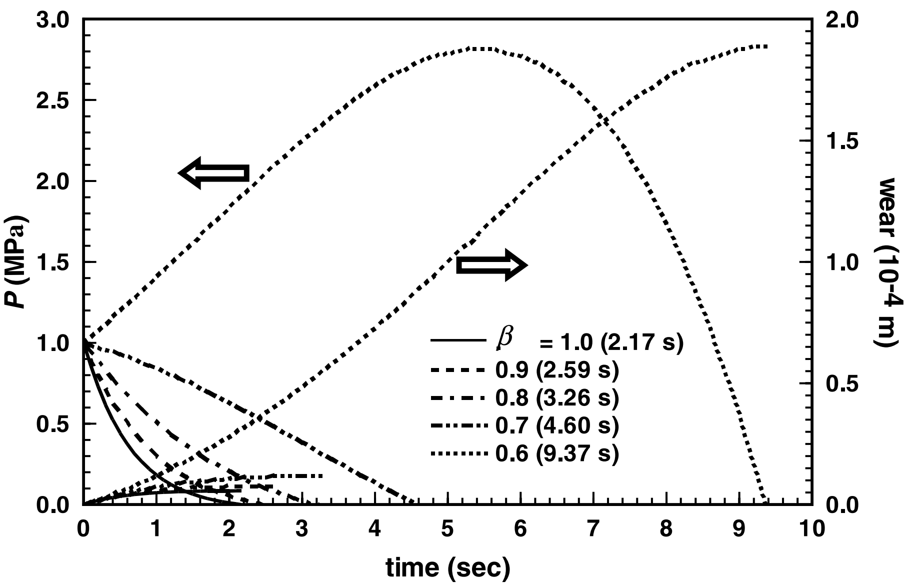

Fig. 7. Contact pressure and wear versus time using various values of β.

4.2. Case 2: constant contact pressure

Since the applied pressure is constant, the wear rate is constant according to Eq. (5). Thus the calculation of the amount of wear is straightforward and omitted here. The end temperature change found using

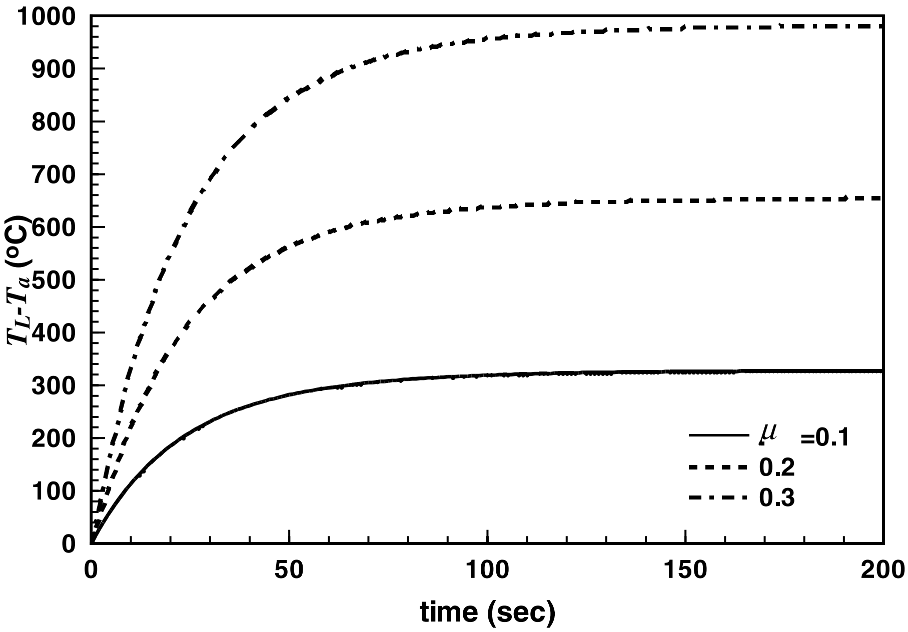

Fig. 8. End-temperature change $T_L - T_a$ versus time using various values of μ and constant pressure of 1 MPa.

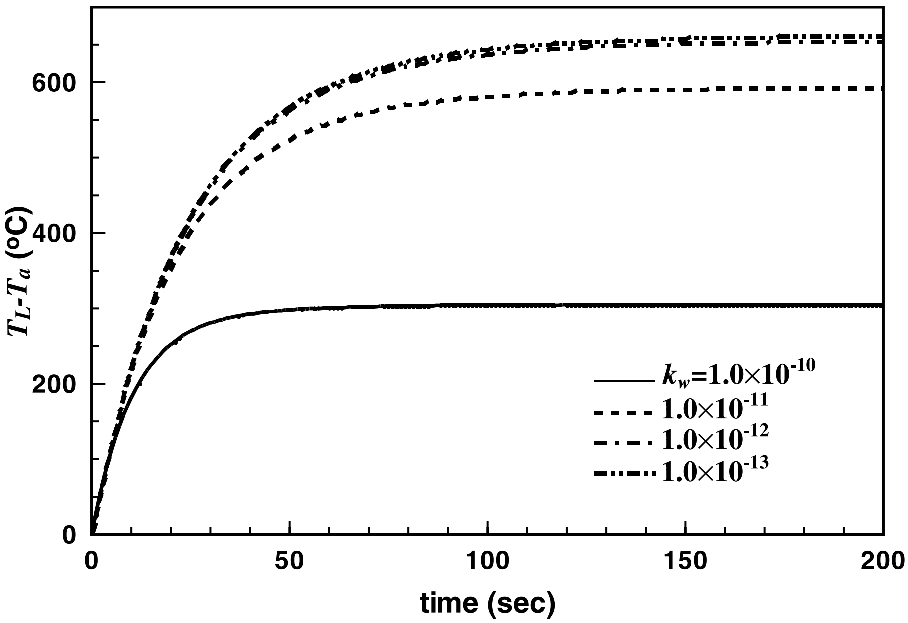

Fig. 9. End-temperature change $T_L - T_a$ versus time using various values of k_w and constant pressure of 1 MPa.

various values of μ is shown in Fig. 8. All three curves show a stabilizing trend. The effect of the wear constant k_w on the end temperature is shown in Fig. 9. It is seen that the difference in $T_L - T_a$ becomes negligibly noticeable between the cases using $k_w = 1.0 \times 10^{-13}$ and 1.0×10^{-14} Pa^{-1}.

5. Conclusion

This paper presents a model and the finite element analysis of the heat conduction of an elastic beam due to frictional contact of a moving rigid object at the end of the beam. During sliding contact, two processes take place. First, the friction shear force generates energy flowing into the beam and the sliding solid. Second, the wear shortens the beam. The temperature increase and the wear in the beam interact with the contact pressure at the contacting end of the beam. Two cases are considered: one with constant contact pressure in which the sliding contact object is allowed to move horizontally as the beam shortens; the other with time-varying contact pressure in which the sliding contact object is held stationary in the horizontal direction. This nonlinear problem is solved using incremental and iterative finite element method where implicit time marching scheme is implemented. Numerical results using various parameters are shown. A C++ computer program is written for the numerical method presented and run on a Pentium II 400 MHz PC. For each case shown in the paper, the computing time is less than 10 s. It is found that

(a) the energy leaving the system due to wear is negligibly small when compared with the energy generated by friction;
(b) the wear life, the time needed for the contact pressure to reduce to zero, of the beam increases as the coefficient of kinetic friction increases;
(c) if the contact pressure is allowed to vary the end-temperature dips after reaching the peak and before exhausting the wear life; on the other hand the end-temperature stabilizes if the contact pressure is held constant.

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